1. In questions 1.1 to 1.7 below one or more of the alternatives are correct. Write the code letter(s), (a), (b), (c), (d) corresponding to the correct alternative(s) in the answer book.

1.1 The eigenvector(s) of the matrix \( \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \), if \( a \neq 0 \), is (are)

(a) \((0, 0, a)\)  
(b) \((a, 0, 0)\)  
(c) \((0, 0, 1)\)  
(d) \((0, a, 0)\)

1.2 The differential equation, \( \frac{d^3 y}{dx^3} + \frac{dy}{dx} + \sin y = 0 \), is

(a) linear  
(b) non-linear  
(c) homogeneous  
(d) of degree two

1.3 Simpson's rule for integration gives exact result when \( f(x) \) is a polynomial of degree

(a) 1  
(b) 2  
(c) 3  
(d) 4

1.4 Which of the following is (are) valid FORTRAN 77 statement(s)?

(a) DO 131 = 1  
(b) A = DIM 7  
(c) READ = 15.0  
(d) GOTO 3 = 10

1.5 Fourier series of the periodic function (period 2\( \pi \)) defined by

\[ f(x) = \begin{cases} 0 & \text{for } -\pi < x < 0 \\ 0 & \text{for } 0 < x < \pi \end{cases} \]

\[ \frac{\pi}{4} + \sum_{n=1}^{\infty} \left[ -\frac{1}{n^2} \cos \frac{n\pi}{2} - \frac{1}{n}\cos \frac{n\pi}{2} \sin \frac{n\pi}{2} \right] \]

By putting \( x = \pi \) in the above, one can deduce that the sum of the series

\[ 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots \]

is

(a) \( \frac{\pi^2}{4} \)  
(b) \( \frac{\pi^2}{6} \)  
(c) \( \frac{\pi^2}{8} \)  
(d) \( \frac{\pi^2}{12} \)

1.6 Which of the following improper integrals is (are) convergent?

(a) \( \int_{0}^{\infty} \frac{\sin x}{1 - \cos x} \, dx \)  
(b) \( \int_{0}^{\infty} \frac{\cos x}{1 + x} \, dx \)

(c) \( \int_{0}^{\infty} \frac{x^2}{1 + x^2} \, dx \)  
(d) \( \int_{0}^{\infty} \frac{1 - \cos x}{x^{3/2}} \, dx \)

1.7 The function \( f(x,y) = x^2 y - 3xy + 2y + x \) has

(a) no local extremum  
(b) one local minimum but no local maximum  
(c) one local maximum but no local minimum  
(d) one local minimum and one local maximum

2. In questions 2.1 to 2.10 below, each blank (_______) is to be suitably filled in. In the answer book write the question number and the answer only. Do not copy the question. Also no explanations for the answers are to be given.

2.1 \( \lim_{x \to 0} \frac{x(e^x - 1) + 2(\cos x - 1)}{x(1 - \cos x)} \) is

2.2 The radius of convergence of the power series

\[ \sum_{n=0}^{\infty} \frac{(3n)!}{(3n)!} x^{3n} \] is ________.

2.3 If the linear velocity \( \dot{v} \) is given by \( \dot{v} = x^2 y \dot{i} + x y z \dot{j} - y z^2 \dot{k} \)

the angular velocity \( \vec{\omega} \) at the point (1, 1, -1) is ________.

2.4 Given the differential equation \( \ddot{y} = x - y \) with the initial condition \( y(0) = 0 \). The value of \( y(0.1) \) calculated numerically up to the third place of decimal by the second order Runge-Kutta method with step size \( h = 0.1 \) is ________.

2.5 For \( X = 4.0 \), the value of 1 in the FORTRAN 77 statement

\[ I = -2 + 2.5 \times X/X + 3 + \frac{3}{4} \] is ________.

2.6 The value of the double integral \( \int_{0}^{1} \int_{0}^{1} \frac{x}{1 + y^2} \, dx \, dy \) is ________.
2.7 If \[ A = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & i & i \\ 0 & 0 & 0 & -i \end{pmatrix} \]

the matrix \( A^4 \), calculated by the use of Cayley - Hamilton theorem or otherwise, is

2.8 Given, \( V = x \cos^2 y \hat{j} + x^2 \hat{i} + z \sin^2 y \hat{k} \)
and S the surface of a unit cube with one corner at the origin and edges parallel to the coordinate axes, the value of the integral 
\[ \int \int_S \nabla \cdot \vec{n} \, dS \]
is ________.

2.9 The differential equation \( y'' + vy = 0 \) is subjected to the boundary conditions 
\( y(0) = 0 \), \( y(\lambda) = 0 \)
In order that the equation has non-trivial solution (s), the general value of \( \eta \) is ________.

2.10 The Laplace transform of the periodic function \( f(t) \) described by the curve below, i.e.
\[ f(t) = \begin{cases} \sin t & \text{if} \ (2n-1) \pi \leq t \leq 2n\pi (n = 1, 2, 3, \ldots) \\ 0 & \text{otherwise} \end{cases} \]
is ________.

3.2. The temperature of an ideal gas is held constant while its volume is increased. The pressure exerted by the gas on the walls of the container decreases because its molecules
(a) strike the walls with smaller force
(b) strike the walls with lower velocities
(c) strike the walls less frequently
(d) collide with each other more frequently

3.3. Although a laser beam is highly directional, its beam width increases with propagation. This increase is due to:
(a) coherence (b) diffraction
(c) polarization (d) interference

3.4 A plane electromagnetic wave of the form
\[ \vec{E} = \vec{E}_0 \cos 2\pi (5 \times 10^{14} \text{ sec}^{-1})t - (2.5 \times 10^6 \text{m}^{-1})x \]
(where \( E_0 \) is a constant and \( y \) is the unit vector along \( y \)-direction) represents a wave propagating along
(a) + x direction (b) + y direction
(c) - x direction (d) - y direction

3.5. While you are listening to a programme from a radio, if a near-by electric light bulb is switched on or switched off, you hear a momentary noise in your radio. This is due to electromagnetic radiation emitted by ________.

3.6. Nuclear fusion reactions require very high temperatures so as to overcome.
(a) nuclear forces (b) van der waals forces
(c) coulomb forces (d) gravitational forces

3.7. In radioactive decay, the disintegration rate of the nuclei is
(a) constant at all times
(b) inversely proportional to half-life of the nuclei
(c) inversely proportional to the number of nuclei at any time
(d) directly proportional to the number of nuclei at any time

3.8. In an hydrogen atom 10.2. eV is given out as radiation when an electron is de-excited to the ground state. The principal quantum number of the excited state is ________.

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**SECTION B : PHYSICS**

3. In the following questions 3.1 to 3.17 there are some multiple-choice questions and some questions where blanks are to be filled in. Answer ALL the questions. All multiple-choice questions have ONE or MORE correct answers those suggested.

**Useful Data :** \( h = 6.63 \times 10^{-34} \text{ Js} \), \( c = 3 \times 10^8 \text{ m/s} \)

3.1 Two particles of masses \( M_1 \) and \( M_2 \) (\( M_1 > M_2 \)) attract each other with a force inversely proportional to the square of the distance between them. The particles are initially at rest and then released. The centre of mass relative to a stationary observer
(a) moves towards \( M_1 \)
(b) move towards \( M_2 \)
(c) remains at rest
(d) moves with a speed proportional to \( \sqrt{\frac{M_1}{M_2}} \)
3.9. Typical current voltage characteristic of a solar cell is given in the following figure by

(a) curve A    (b) curve B
(c) curve C    (d) curve D

3.10. Consider a solid sphere and a hollow sphere, both of mass M, radius R and initially at rest, which start rolling down the same inclined plane without slipping. At the bottom of the inclined plane, the ratio of speeds \( v_{\text{solid}} / v_{\text{hollow}} \) is

(a) 1    (b) \( \sqrt{12} / \sqrt{7} \)
(c) \( \sqrt{10} / \sqrt{7} \)    (d) \( \sqrt{25} / \sqrt{21} \)

[Note: The moment of inertia about any diameter for a solid sphere is \( (2/5) MR^2 \), and for a hollow sphere \( (2/3) MR^2 \)]

3.11. An optical fibre consists of a cylindrical dielectric rod of refractive index \( n^1 \), surrounded by another dielectric of refractive index \( n^2 \), where \( n^2 < n^1 \), as shown in the following figure. If a ray is incident from air at an angle \( \theta \) to the axis, then it undergoes total internal reflection at the interface AB if

(x) \( \sin \theta > \sin \theta_0 \)
(b) \( \sin \theta < \sin \theta_0 \)
(c) \( \sin \theta > \sin \theta_0 \)
(d) \( \sin \theta < \sin \theta_0 \)

3.12. For a uniformly charged sphere of radius \( R \) and charge density \( \rho \), the ratio of magnitude of electric fields at distances \( R/2 \) and \( 2R \) from the centre, i.e.,

\[ \frac{E(r = R/2)}{E(r = 2R)} \] is

A. 1
B. 2
C. 4
D. 8

3.13. A long solenoid of radius \( R \), and having \( N \) turns per unit length carries a time dependent current \( I(t) = I_0 \cos(\omega t) \). The magnitude of induced electric field at a distance \( R/2 \) readily from the axis of the solenoid is

(a) \( \frac{R}{2} \mu_0 NI_0 \omega \sin(\omega t) \)
(b) \( \frac{R}{2} \mu_0 NI_0 \omega \cos(\omega t) \)
(c) \( \frac{R}{2} \mu_0 NI_0 \omega \sin(\omega t) \)
(d) \( \frac{R}{2} \mu_0 NI_0 \omega \cos(\omega t) \)

3.14. In an electron diffraction experiment, planes of a crystal with spacing 1 A\(^\circ\) between them yield the first maximum at a Bragg angle of \( \theta = 30^\circ \). The momentum of the electrons is \( \text{_________} \) eV m/s.

3.15. A conventional unit cell of close packed face centered cubic (FCC) structure made up of hard spheres has a cube edge of a A\(^\circ\). The radius of the sphere is _______ A\(^\circ\).

3.16. A light beam of frequency \( 1.2 \times 10^{15} \text{ Hz} \) is incident on a metal in a photoelectric effect experiment. The corresponding maximum kinetic energy of the ejected photoelectrons from the metal is \( 6.63 \times 10^{-19} \text{ J} \). The characteristic cut-off frequency of the metal is _______ Hz.

3.17. Consider the semiconductors A and B. The figure shows variation of In \( p \) with \( 1/K \), where \( p \) is resistivity and \( T \) the temperature, for the two semiconductors. Choose the correct statements (s).

\[ \text{In } p v.s. \frac{1}{(K^{-1})} \]

(a) the bandgap energy of A is larger than that of B.
(b) the bandgap energy of A is smaller than that of B.
(c) the maximum wavelength of light needed to create an electron hole pair is larger in A than in B.
(d) the maximum wavelength of light needed to create an electron hole pair is smaller in A than in B.
4. While starting a dc shunt motor:
(a) reduced armature voltage \( V_a \) and reduced field voltage \( V_f \) should be applied and full regulator resistance \( R_s \) should be included in the field circuit.
(b) reduced \( V_f \) but full \( V_a \) should be applied and \( R_s \) should be zero.
(c) rated \( V_a \) and rated \( V_f \) should be applied and \( R_s \) should be zero.
(d) rated \( V_a \) and rated \( V_f \) should be applied and \( R_s \) should be maximum.

4.5. A 6 pole 3 phase wound-rotor induction machine is driven by another machine at 180 rpm. The rotor of the induction machine is connected to a 50 Hz system. If the mechanical rotation of the rotor is in the same direction as the rotor winding flux rotation, then the frequency of the stator voltage will be
(a) 50 Hz
(b) 140 Hz
(c) 150 Hz
(d) 200 Hz.

4.6. For the amplifier circuit of the figure is the transistor a \( B \) of 800. The midband voltage gain \( V^0 / V^1 \) of the circuit will be
(a) 0
(b) < 1
(c) \approx 1
(d) 800

4.7. The waveshape of \( V^0 \) in the figure will be
4.8. For the logic circuit shown in the figure is the output $Y$ is equal to

(a) $\overline{ABC}$
(b) $\overline{A} + \overline{B} + \overline{C}$
(c) $\overline{AB} + \overline{BC} + \overline{A} + \overline{C}$

4.9. In the figure, is in ideal moving iron voltmeter $M$ will read

(a) 7.07 V  
(b) 12.24 V  
(c) 14.14 V  
(d) 20.0 V

**PART - II**

Question 6 consists of 25 subquestions each carrying 2 marks, All the subquestions must be answered in the sequence in which they appear. In the multiple choice questions, four alternatives, a,b, c and d are given, of which more than one may be correct.

6.1. A network contains linear resistors and ideal voltage sources. If values of all the resistors are doubled, then the voltage across each resistor is

(a) halved  
(b) doubled  
(c) increased by four times  
(d) not changed

6.2. The two electrical sub network $N_1$ and $N_2$ are connected through three resistors as shown in the figure. The voltage across 5 ohm resistor and 1 ohm resistor are given to be 10 V and 5V, respectively. Then voltage across 15 ohm resistor is

(a) $-105$ V  
(b) $+105$ V  
(c) $-15$ V  
(d) $+15$ V

6.3. In the figure is $A^1$, $A^2$ and $A^3$ are ideal ammeters. If $A^1$ reads 5A, $A^2$ reads 12 A, then $A^3$ should read.

(a) 7 A  
(b) 12 A  
(c) 13 A  
(d) 17 A

6.4. If $F(s) = [f(t)] = \frac{K}{(s+1)(s^2 + 4)}$ then $\lim_{t \to \infty} f(t)$ is given by

(a) $K/4$  
(b) zero  
(c) infinite  
(d) undefined

6.5. If $s^3 + 3s^2 + 4s + A = 0$, then all the roots of this equation are in the left half plane provided that

(a) $A > 12$  
(b) $-3 < A < 4$  
(c) $0 < A < 12$  
(d) $5 < A < 12$

6.6. The built-in potential (Diffusion Potential) in a p-n junction

(a) is equal to the difference in the Fermi level of the two sides, expressed in volts  
(b) increases with the increase in the doping levels of the two sides  
(c) increases with the increase in temperature  
(d) is equal to the average of the Fermi levels of the two sides

6.7. $\alpha$ - cut off frequency of a bipolar junction transistor

(a) increases with the increase in base width  
(b) increases with the increase in emitter width  
(c) increases with increase in the collector width  
(d) increases with decrease in the base width.

6.8. Negative feedback in amplifiers

(a) improves the signal to noise ratio at the input  
(b) improves the signal to noise ratio at the output  
(c) does not affect the signal to noise ratio at the output  
(d) reduces distortion
6.9. In a multi-stage R-C coupled amplifier the coupling capacitor
(a) limits the low frequency response
(b) limits the high frequency response
(c) does not affect the frequency response
(d) blocks the d.c. component without affecting the frequency response

6.10. The bandwidth of an n-stage tuned amplifier, with each stage having a bandwidth of $B_0$, is given by
(a) $B_0/n$
(b) $B_0/\sqrt{n}$
(c) $B_0 2^{1/n} - 1$
(d) $B_0/2^{1/n} - 1$

6.11. In a transistor push-pull amplifier
(a) there is no d.c. component in the output
(b) there is no distortion in the output
(c) there are no even harmonics in the output
(d) there are no odd harmonics in the output

6.12. 2's complement representation of a 16-bit number (one sign bit and 15 magnitude bits) is FF11. Its magnitude in decimal representation is
(a) 0
(b) 1
(c) 32,767
(d) 65,535

6.13. Boolean expression for the output of XNOR (Equivalent) logic gate with inputs A and B is
(a) $A \bar{B} + \bar{A} B$
(b) $\bar{A} \bar{B} + AB$
(c) $(A + B)(A + \bar{B})$
(d) $(\bar{A} + \bar{B})(A + B)$

6.14. Consider the circuits shown in the figure is if the diode used here has the V-I characteristic as in
the figure is then the output wave form $v_o$ is

6.15. A pulse train with a frequency of 1 MHz is counted using a modulo 1024 ripple-counter built with J-K flip flops. For proper operation of the counter, the maximum permissible propagation delay per flip flop stage is . . . . . . n Sec.

6.16. In a microprocessor, the register which holds the address of the next instruction to be fetched is
(a) Accumulator
(b) Program Counter
(c) Stack Pointer
(d) Instruction Register

6.17. In a microcomputer, wait states are used to
(a) make the processor wait during a DMA operation
(b) make the processor wait during an interrupt processing
(c) make the processor wait during a power shutdown
(d) interface slow peripherals to the processor

6.18. The function shown in figure can represent a probability density function for $A$ . . . . .

6.19. Which of the following demodulator(s) can be used for demodulating the signal
\[ x(t) = 5(1 + 2 \cos 2000 \pi t) \cos 2000 \pi t. \]
(a) Envelope demodulator
(b) Square-law demodulator
(c) Synchronous demodulator
(d) None of these

6.20. A superheterodyne radio receiver with an intermediate frequency of 455 KHz is tuned to a station operating at 1200 KHz. The associated image frequency is . . . . . . KHz.

6.21. Sketch the waveform (with properly marked axes) at the output of a matched filter matched for a signal $S(t)$, of duration $T$, given by
\[ S(t) = \begin{cases} A & \text{for } 0 < t < \frac{2}{3}T \\ 0 & \text{for } \frac{2}{3}T \leq t < T \end{cases} \]

6.22. Six independent low pass signals of bandwidth $3$ W, $W$, $W$, $2W$, $3W$, and $2W$ are to be time-division-multiplexed on a common channel using PAM. To achieve this, the minimum transmission bandwidth of the channel should be . . . . . Hz.
6.23. A material is described by the following electrical parameters as a frequency of 10 GHz, \( \sigma = 10^6 \) mho/m, \( \mu = \mu_0 \) and \( \sigma / \sigma_0 = 10 \). The material at this frequency is considered to be

\[
\sigma_0 = \frac{1}{36\pi} \times 10^{-9} \text{ F/m}
\]

(a) a good conductor
(b) a good dielectric
(c) neither a good conductor, nor a good dielectric
(d) a good magnetic material

6.24. Consider a transmission line of characteristic impedance of 50 ohm. Let it be terminated at one end by + j 50 ohm. The VSWR produced by it in the transmission line will be

(a) + 1 (b) 0
(c) \( \infty \) (d) + j

6.25. A plane wave is incident normally on a perfect conductor as shown in figure. Here \( E'_x = E' \) and \( P \) are electric field, magnetic field and Poynting vector respectively, for the incident wave. The reflected wave should be

\( (a) E'_x = -E' \) (b) \( H_y = -H_y \)
\( (c) \vec{P} = -\vec{P} \) (d) \( E'_x = E_x \)

ANSWERS

6. 1 (d) 6. 2 (a) 6. 3 (c) 6. 4 (c) 6. 5 (c) 6. 6 (n,b,c) 6. 7 (d) 6. 8 (b,d)
6. 9 (a) 6. 10 (c) 6. 11 (a,c) 6. 12 (c) 6. 13 (c) 6. 14 (c) 6. 15 (*) 6. 16 (b)
6. 17 (d) 6. 18 (*) 6. 19 (a,b,c) 6. 20 (*) 6. 21 (*) 6. 22 (*) 6. 23 (n) 6. 24 (c)

6. 25 (b,c,d)